Destroying extremal Kerr-Newman black holes with test particles

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Abstract

It has been shown that a nearly extremal black hole can be overcharged or overspun by a test particle if radiative and self-force effects are neglected, indicating that the cosmic censorship might fail. In contrast, the existing evidence in literature suggests that an extremal black hole cannot be overcharged or overspun in a similar process. In this paper, we show explicitly that even an exactly extremal black hole can be destroyed by a test particle, leading to a possible violation of the cosmic censorship. By considering higher order terms, which were neglected in previous analysis, we show that the violation is generic for any extremal Kerr-Newman black hole with nonvanishing charge and angular momentum. We also find that the allowed parameter range for the particle is very narrow, indicating that radiative and self-force effects should be considered and may prevent violation of the cosmic censorship.

1 Introduction

If a singularity is not covered by a black hole horizon, it can be seen by distant observers and is called a naked singularity. The weak "cosmic censorship" conjecture states that naked singularities cannot be formed by gravitational collapse with physically reasonable matter [1]. A precise statement of this conjecture was given in [2]. Although a general proof of this conjecture has not been given, evidence in favor of it has been found and discussed in the past few decades. One way of testing the cosmic censorship conjecture is to see whether the black hole horizon can be destroyed by an object falling into the black hole. In the seminal work, Wald [3] proved that a test particle cannot destroy the horizon of an extremal

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black hole. This work has been revisited and extended by a number of authors in the last decade [4]-[8]. There were two crucial assumptions in Wald's treatment. Firstly, the existing black hole is extremal. Secondly, only linear terms in particle's energy, charge and angular momentum are kept in the analysis. By releasing the two assumptions, Hubeny showed that a nearly extremal Reissner-Nordstrom black hole can be overcharged by a test particle. Recently, Jacobson showed that a nearly Extremal Kerr black hole can be overspinned. These results apparently indicate violations of the cosmic censorship. At least, they point out the test particle assumption may not be valid and the radiative and self-force effects should be considered.

Note that the results in [4, 6] agree with Wald's in the extremal limit. So it seems that the cosmic censorship holds anyway when one tries to overcharge or overspin extremal black holes. However, an overlooked fact is that the authors of [4] and [6] only considered RN black hole and Kerr black hole respectively, while Wald considered the combination, i.e., the Kerr-Newman (KN) black hole. To distinguish from RN and Kerr solutions, we shall refer to KN black holes as those with nonvanishing charge and angular momentum. By reexamining Wald's arguments, we find that counter examples can be found if higher order terms are included in the calculation (High order terms have been considered in [4, 6] for RN and Kerr black holes, but causing no violation to the cosmic censorship in the extremal cases). This means that the cosmic censorship is not safe even for extremal black holes. We further find that the allowed range of the particle's energy is very small, which means that the particle's parameters must be finely tuned. This suggests that radiation and self-force effects are necessary for a complete proof of the cosmic censorship.

2 Review of Wald's proof

In this section, we review the gedanken experiment in extremal charged Kerr black holes proposed by Wald[3]. Consider the charged Kerr solution

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 + 2g_{t\phi}dtd\phi\,. \tag{1}$$

Assume the vector potential is in the form

$$A_a = A_t dt_a + A_\phi d\phi_a\,. \tag{2}$$

A charged particle with mass m and charge q moves in the spacetime with four-velocity

$$u^a = \dot{t}\left(\frac{\partial}{\partial t}\right)^a + \dot{r}\left(\frac{\partial}{\partial r}\right)^a + \dot{\theta}\left(\frac{\partial}{\partial \theta}\right)^a + \dot{\phi}\left(\frac{\partial}{\partial \phi}\right)^a\,. \tag{3}$$

The conserved energy and angular momentum are

$$\begin{aligned} E &= -t^a(mu_a + qA_a)\,, && (4)\\ L &= \phi^a(mu_a + qA_a)\,. && (5) \end{aligned}$$

Solving Eqs. (4) and (5) for $\dot{t}$ and $\dot{\phi}$, we have

$$\begin{aligned}
\dot{t} &= \frac{Eg_{\phi\phi} + g_{t\phi}L + A_t g_{\phi\phi} q - A_\phi g_{t\phi} q}{m(g_{t\phi}^2 - g_{\phi\phi} g_{tt})}\,, \qquad (6)\\
\dot{\phi} &= -\frac{Eg_{t\phi} + g_{tt}L + A_t g_{t\phi} q - A_\phi g_{tt} q}{m(g_{t\phi}^2 - g_{\phi\phi} g_{tt})}\,. \qquad (7)
\end{aligned}$$

Substituting the two formulae into

$$g_{ab}u^a u^b = -1 \qquad (8)$$

and solve the quadratic equation for E, we find

$$\begin{aligned}
E &= \frac{-g_{t\phi}L - qA_t g_{\phi\phi} + qA_\phi g_{t\phi}}{g_{\phi\phi}}\\
&\pm \frac{1}{g_{\phi\phi}}\sqrt{(g_{t\phi}^2 - g_{\phi\phi}g_{tt})[L^2 - 2qLA_\phi + q^2A_\phi^2 + m^2 g_{\phi\phi}(1 + g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2)]}\\
&. \qquad (9)
\end{aligned}$$

Noting that u^a is future pointing which implies $\dot{t} > 0$. Therefore, we should take the plus sign in front of the square root in Eq. (9). Consequently,

$$E \geq \frac{-g_{t\phi}L - qA_t g_{\phi\phi} + qA_\phi g_{t\phi}}{g_{\phi\phi}}\,. \qquad (10)$$

The Kerr-Newmann metric is given by [9]

$$\begin{aligned}
g_{tt} &= -\frac{\Delta - a^2\sin^2\theta}{\Sigma}\,, \qquad (11)\\
g_{t\phi} &= -\frac{a\sin^2\theta(r^2 + a^2 - \Delta)}{\Sigma}\,, \qquad (12)\\
g_{\phi\phi} &= \frac{(r^2 + a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\sin^2\theta\,, \qquad (13)\\
A_t &= -\frac{Qr}{\Sigma}, \quad A_\phi = \frac{Qr}{\Sigma}a\sin^2\theta\,, \qquad (14)\\
g_{rr} &= \frac{\Sigma}{\Delta}\,, \qquad (15)\\
g_{\theta\theta} &= \Sigma\,, \qquad (16)
\end{aligned}$$

with

$$\begin{aligned}
\Sigma &= r^2 + a^2\cos^2\theta\,, \qquad (17)\\
\Delta &= r^2 + a^2 + Q^2 - 2Mr\,. \qquad (18)
\end{aligned}$$

Then at the horizon $r = r_+$, Eq. (9) is written as

$$E = \frac{aL + qQr}{a^2 + r^2} + m\sqrt{\frac{(a^2 + 2r_+^2 + a^2\cos^2(2\theta))^2}{4(a^2 + r_+^2)^2}\dot{r}^2}\,, \qquad (19)$$

and thus

$$E \geq \frac{aL + qQr}{a^2 + r^2} . \tag{20}$$

For an extremal black hole $r_+ = M$, we have

$$E \geq \frac{aL + qQM}{a^2 + M^2} . \tag{21}$$

On the other hand, to destroy the black hole horizon with $M^2 = Q^2 + a^2$, the particle must satisfy

$$(E + M)^2 < (Q + q)^2 + \left(\frac{aM + L}{M + E} \right)^2 . \tag{22}$$

Expand the last term around $E = 0$, we have

$$E^2 + M^2 + 2ME < Q^2 + q^2 + 2qQ - \frac{(L + aM)^2}{M^2} + \frac{2(L + aM)^2 E}{M^3} . \tag{23}$$

Using $M^2 = Q^2 + a^2$ and keeping the terms linear to q, E, L, we have

$$E < \frac{aL + MqQ}{M^2 + a^2} , \tag{24}$$

which contradicts Eq. (21). Thus, the cosmic censorship is upheld if higher order terms are neglected. In the next section, we shall see that higher order terms do not change this result if one attempts to destroy an extremal Kerr or RN black hole.

3 Kerr and RN cases

The above result is derived from a Kerr-Newman black hole. Now let us consider two reduced cases.

1. Pure Kerr ($Q = q = 0, M = a$)

Eq. (21) reduces to

$$E \geq \frac{L}{2M} , \tag{25}$$

and Eq. (22) reduces to

$$E + M < \frac{M^2 + L}{E + M} , \tag{26}$$

i.e.,

$$E^2 + 2ME < L , \tag{27}$$

$$E < \frac{L}{2M} - \frac{E^2}{2M} < \frac{L}{2M}\,, \tag{28}$$

So no solution can be found.

2. Pure RN($a = L = 0, M = Q$)

Eq. (21) reduces to

$$E \geq q\,, \tag{29}$$

and Eq. (22) reduces to

$$E + M < Q + q\,, \tag{30}$$

i.e.,

$$E < q\,. \tag{31}$$

Obviously, there is no solution.

Thus, there is no violation of cosmic censorship for either Kerr black hole or RN black hole, agreeing with the results of Jacobson and Hubeny. Differing from the treatment in Section 2, no linear approximation has been made in the above proof.

4 Violation of the cosmic censorship for extremal KN black holes

From the last section, we see that the cosmic censorship conjecture has passed the test of gedanken experiments in extremal RN or Kerr black holes, even without linear approximation. However, it is unknown whether a complete analysis can lead to a different conclusion for extremal KN black holes ($Q \neq 0$ and $a \neq 0$). We first show that the two inequalities (21) and (22) can be simplified and combined into one. Define

$$W = (M + E)^2\,, \tag{32}$$

and rewrite Eq. (22) as

$$W^2 - (Q + q)^2 W - (aM + L)^2 < 0\,. \tag{33}$$

This means

$$W_1 < W < W_2 \tag{34}$$

with

$$W_{1,2} = \frac{(Q+q)^2 \pm \sqrt{(Q+q)^4 + 4(aM+L)^2}}{2}\,. \tag{35}$$

From Eq. (21) we have

$$W > \left(\frac{aL + qQM}{a^2 + M^2} + M\right)^2 \equiv W_3\,. \tag{36}$$

Obviously, $W_1 < 0$ and $W_2, W_3 > 0$. Therefore, the necessary and sufficient condition for both inequalities (21) and (22) being satisfied is

$$W_2 > W_3\,, \tag{37}$$

i.e.,

$$\begin{aligned} s &\equiv W_2 - W_3 && (38)\\ &= \frac{(Q+q)^2 + \sqrt{(Q+q)^4 + 4(aM+L)^2}}{2} - \left(\frac{aL+qQM}{a^2+M^2} + M\right)^2 && (39)\\ &> 0\,. && (40)\end{aligned}$$

Expand the right-hand side to the second order in q and L, we find

$$\frac{2a^2M^2(3M^2-a^2)}{(a^2+M^2)^3}q^2 + \frac{M^2(-3a^2+M^2)}{(a^2+M^2)^3}L^2 - \frac{2aMQ(3M^2-a^2)}{(a^2+M^2)^3}qL > 0\,. \tag{41}$$

Now we can estimate the allowed range of E. From

$$W_3 < W < W_2\,, \tag{42}$$

we see the allowed range of E, denoted by ΔE, satisfies $2M\Delta E \sim W_2 - W_3$. Then Eq. (41) suggests that ΔE is of order q^2/M or L^2/M^3.

Eq. (41) shows that as long as $Q \neq 0$, $a \neq 0$ and $q \neq 0$, there always exist solutions by taking sufficiently small values of L. To be specific, we choose the parameter set to be $M = 100$, $a = 90$, and then $Q = \sqrt{M^2 - a^2} = 43.6$. We further choose $q = 0.1$ such that the test body condition $q \ll Q$ is met. Now s in Eq. (40) can be treated as a function of L. The plot in Fig. 1 confirms that small values of L always lead to positive s.

For illustration, we take $L = 5$ and find $4.8944 \times 10^{-2} < E < 4.8964 \times 10^{-2}$. So $\Delta E \sim 2 \times 10^{-5}$, which is comparable to $q^2/M = 10^{-4}$ and $L^2/M^3 = 2.5 \times 10^{-5}$, as expected.

5 Conclusions

We have shown that, without taking into account of the radiative and self-force effects, a test particle may destroy the horizon of an extremal charged Kerr black hole, resulting in an apparent violation of the cosmic censorship. The violation is generic for any extremal KN black hole. As shown by Wald [3], there would be no violation if higher order terms are neglected. We also show that the energy of the particle must be finely tuned. A similar

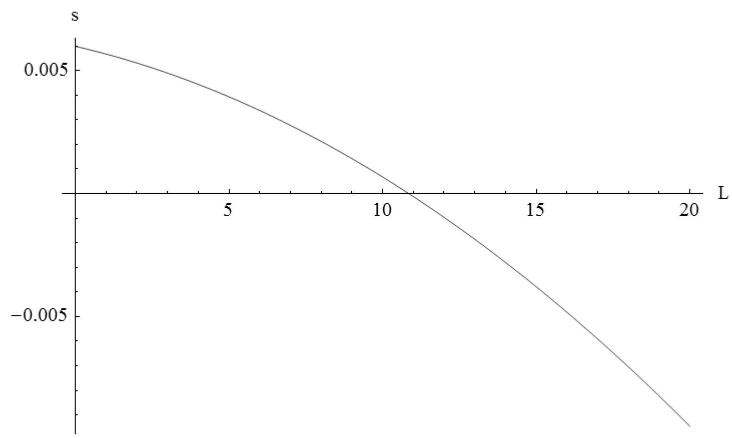

Figure 1: Plot of $s - L$. For small values of L, s is always positive.

fine tuning has been pointed out and discussed in [7] for nearly extremal Kerr black holes, which is a strong indication that radiative and self-force effects should be included. Our results suggest that these effects cannot be neglected even for extremal black holes.

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